Fundamentals of Communications Engineering

Department of Communications Engineering, College of Engineering, University of Diyala, 2016-2017

Class: Second Year
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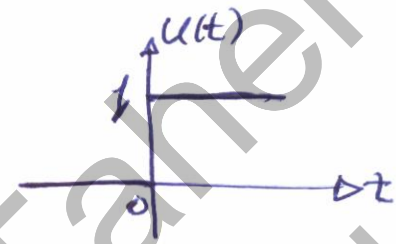
Room: Comm-02

Lecture: 03

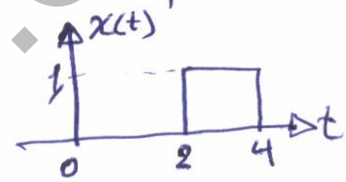
Some Important Signals

□ Unit Step Function $u(t)$

$$u(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases} \qquad (46)$$



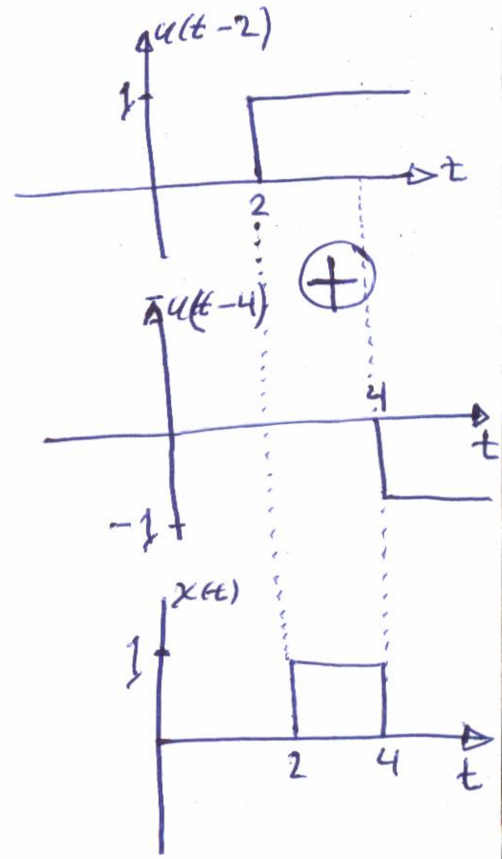
Ex. Describe mathematically, using the step function, the signal shown aside.



Solution

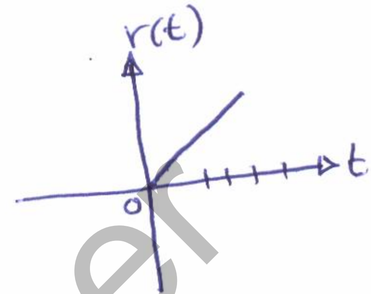
This signal can be constructed from $u(t)$ functions as

$$x(t) = u(t-2) - u(t-4)$$



Ramp Signal

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases} \quad \text{——— (47)}$$



*Note: The ramp function can be found using the unit step function

$$r(t) = \int_{-\infty}^{t} u(\lambda)\, d\lambda \quad \text{——— (48)}$$

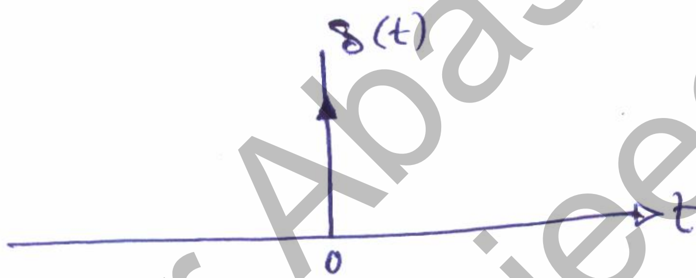
OR the unit step function can be determined from the ramp function as

$$u(t) = \frac{d}{dt} r(t) \quad \text{——— (49)}$$

□ Delta function $\delta(t)$

* It is also called Dirac function.
* OR we call it the impulse signal.

$$\delta(t) = \begin{cases} \text{undefined} & t = 0 \\ 0 & \text{otherwise} \end{cases} \qquad \text{———} (50)$$

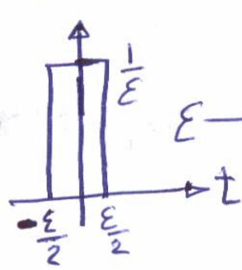


* $\delta(t)$ or delta function is very important in communication systems.

* We can define it again as

$$\left.\begin{aligned} \delta(t) &= 0 \quad t \neq 0 \\ \int_{-\infty}^{\infty} \delta(t) &= 1 \end{aligned}\right\} \text{———} (51)$$

* we can consider $\delta(t)$ as a rectangle function



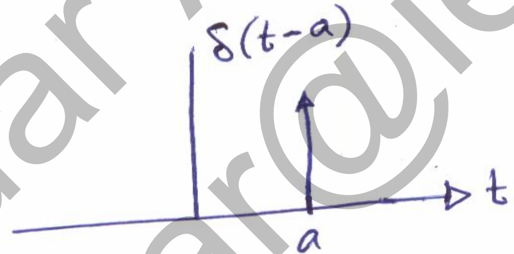
* if a signal $f(t)$ multiplied by $\delta(t)$:-

$$f(t)\,\delta(t) = f(0)\,\delta(t) \quad \text{——— (52)}$$

and note that

$$\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = f(0) \quad \text{——— (53)}$$

* Delta function can also be shifted



then

$$f(t)\,\delta(t-a) = f(a)\,\delta(t-a) \quad \text{——— (54)}$$

and then

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-a)\,dt = f(a) \quad \text{——— (55)}$$

* Delta function is the derivative of the unit step function

$$\delta(t) = \frac{d}{dt} u(t) \qquad (56)$$

<u>Ex.1</u> $f(t) = 8\, e^{-t^3/13} \cos(\omega t)$

then $f(t)\delta(t) = 8\, e^{-t^3/13} \cos(\omega t)\, \delta(t)$

$$= f(0)\,\delta(t)$$

$$= 8\, e^{0} \cos(0)\, \delta(t)$$

$$= 8\, \delta(t)$$

* by integrating $f(t)\delta(t)$ we get

$$\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = \int_{-\infty}^{\infty} 8\, e^{-t^3/13} \cos(\omega t)\, \delta(t)\, dt$$

$$= \int_{-\infty}^{\infty} 8\,\delta(t)\,dt$$

$$= 8 \int_{-\infty}^{\infty} \delta(t)\,dt$$

$$= 8 \qquad \blacksquare$$

EX. evaluate the integral $g(t)=\int_{-\infty}^{\infty} A\cos(t)\,\delta(t-\pi)\,dt$ (31)

Solution $\cos(t)\,\delta(t-\pi) = \cos(\pi)\,\delta(t-\pi)$

$= -1\,\delta(t-\pi)$

$$g(t) = \int_{-\infty}^{\infty} -\delta(t-\pi)\,dt = -1$$ ■

EX. Evaluate $g(t) = \int_{-9}^{2} \cos(t)\,\delta(t-2.1)\,dt$

Solution $\cos(t)\,\delta(t-2.1) = \cos(2.1)\,\delta(t-2.1)$

$$g(t) = \int_{-9}^{2} \underbrace{\cos(2.1)}_{\text{constant}}\,\delta(t-2.1)\,dt = 0$$

$g(t)=0$ because 2.1 is out of the integration range.

* From Equation (54), we conclude the Sampling property of $\delta(t)$ as,

$$\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = f(0)\int_{-\infty}^{\infty} \delta(t)\,dt \quad \text{——— (57)}$$

$$= f(0)$$

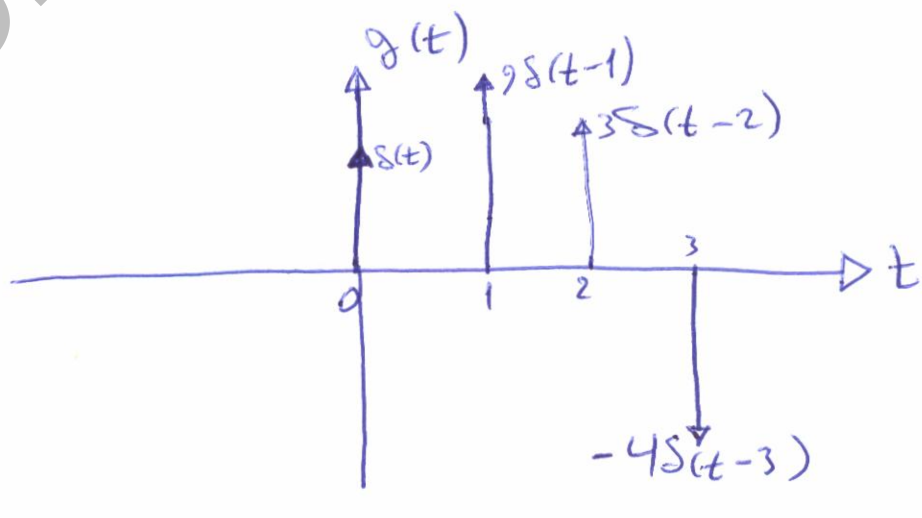
* Sampling property is also called Sifting property

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-a)\,dt = f(a) \quad \text{——— (58)}$$

~~* Furth~~

EX. Draw the function $g(t) = \delta(t) + 9\delta(t-1) + 3\delta(t-2) - 4\delta(t-3)$.

Solution



Scaling property of $\delta(t)$

$$\delta(t) \longrightarrow \delta(t)$$

$$\delta(at) \longrightarrow \frac{1}{|a|}\delta(t)$$

* generally

$$\delta(a(t-t_0)) \longrightarrow \frac{1}{|a|}\delta(t-t_0) \quad (59)$$

Ex. sketch $g(t) = \delta(3t) + \delta\left(\frac{t-1}{2}\right) + \delta\left(\frac{t}{2}-1\right)$

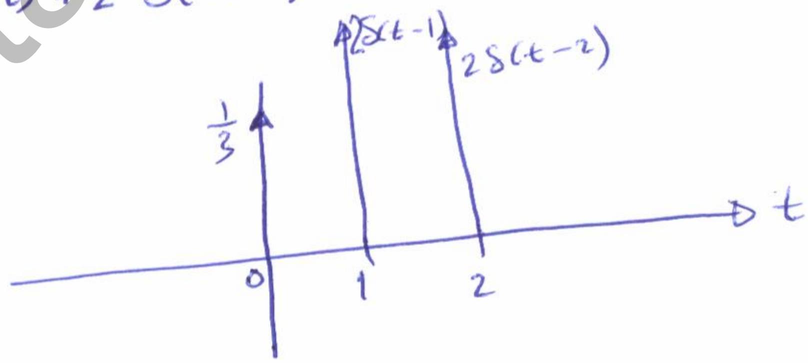
solution

$$\delta(3t) = \frac{1}{3}\delta(t)$$

$$\delta\left(\frac{t-1}{2}\right) = \delta(0.5(t-1)) = 2\delta(t-1)$$

$$\delta\left(\frac{t}{2}-1\right) = \delta\left(\frac{1}{2}(t-2)\right) = 2\delta(t-2)$$

$$\therefore g(t) = \frac{1}{3}\delta(t) + 2\delta(t-1) + 2\delta(t-2)$$

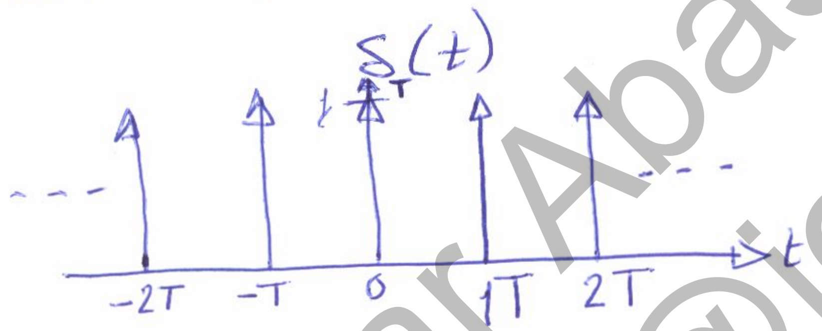


The unit Periodic Impulse

* Also called Impulse Train.

$$\boxed{\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)} \qquad (60)$$

* It is a uniformly spaced infinite sequence of unit impulses.



* If the function $\delta_T(t)$ has been scaled by factor of a then

$$\delta_T(a(t-t_0)) = \sum_{k=-\infty}^{\infty} \delta(a(t-t_0)-kT)$$

Using the scaling property of $\delta(t)$:-

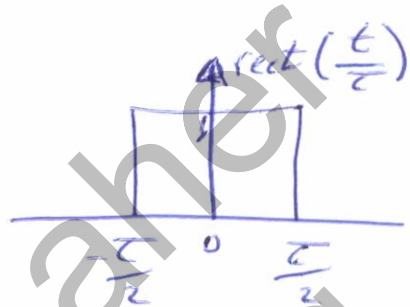
$$\boxed{\delta_T(a(t-t_0)) = \frac{1}{|a|} \sum_{k=-\infty}^{\infty} \delta(t-t_0-kT/a)} \qquad (61)$$

* Now the summation is a periodic impulse of period $\frac{T}{a}$.

$$\delta_T(a(t-t_0)) = \frac{1}{|a|} \delta_{\frac{T}{a}}(t-t_0) \qquad (62)$$

□ Unit rectangle Function $rect(t) = \Pi(t)$

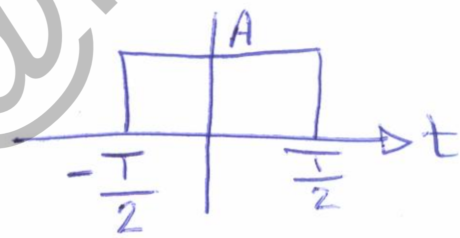
$$rect(t) = \begin{cases} 1 & |t| < \frac{1}{2} \\ 0 & \text{outside} \end{cases} \qquad \text{——— (63)}$$



* Rectangle function also called; Gate function or Box function.

* Unit rectangle function can also represented by $rect(t)$, $\Pi(t)$.

* Total area of $\Pi(t) = 1$

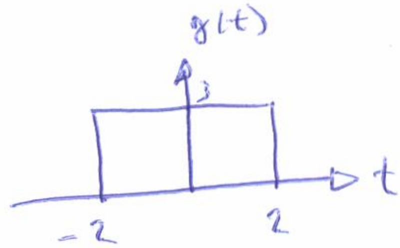


$\therefore$ general form will be

$$\boxed{g(t) = A\, rect\left(\frac{t}{T}\right)} \qquad \text{——— (64)}$$

EX. sketch $g(t) = 3\, rect(0.25t)$.

Solution $\quad A = 3, \quad T = \frac{1}{0.25} = 4 \Rightarrow$



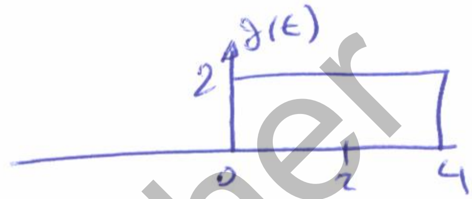
Ex. sketch $g(t) = 2\Pi(0.25(t-2))$.

solution

$A = 2$

$T = 4$

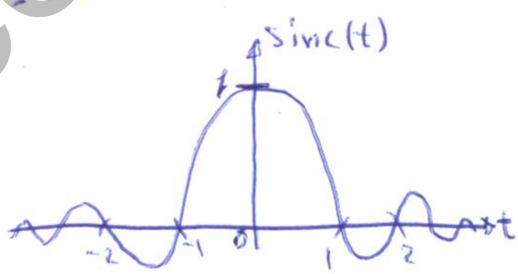
$g(t) \xrightarrow{\text{time shifted}} g(t-2)$



□ Sinc function

* sinc function can be defined as:

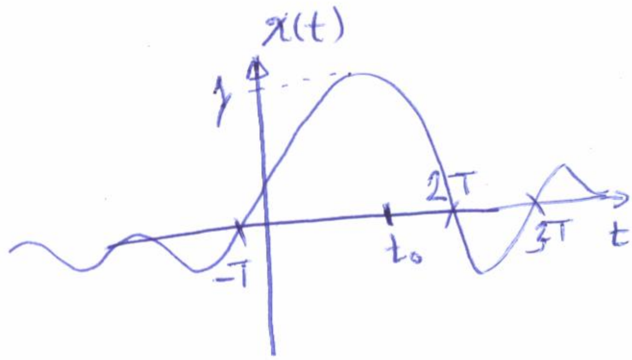
$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t} = \begin{cases} 1 & t = 0 \\ 0 & t = \mp 1, \mp 2, \dots \end{cases} \qquad (65)$$



$$\int_{-\infty}^{\infty} \mathrm{sinc}(t)\, dt = \pi \qquad (66)$$

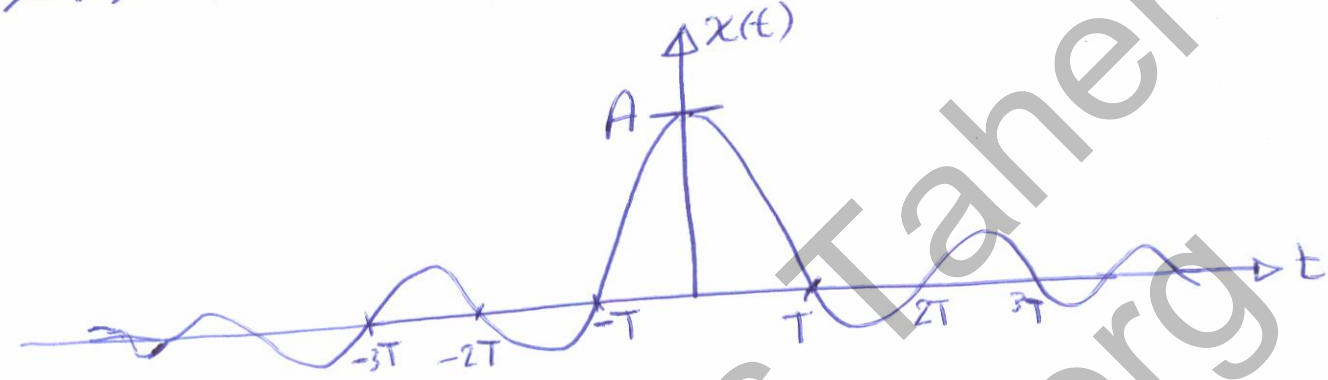
OR $x(t) = \mathrm{sinc}(\omega_0 t) = \mathrm{sinc}(2\pi f_0 t) = \mathrm{sinc}\left(\frac{2\pi}{T_0} t\right)$

Ex. sketch $x(t) = \mathrm{sinc}\left(\frac{t - t_0}{T}\right)$



∘∘ General form of sinc(t) function is

$$x(t) = A\,\mathrm{sinc}\left(\frac{t}{T}\right) \quad \text{——— (67)}$$



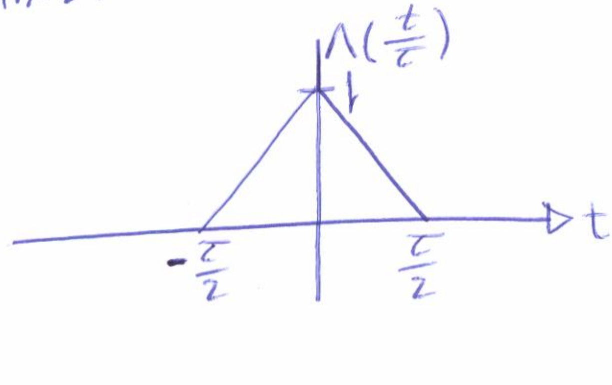
* Sinc function is one of the most important functions in communication systems.

* Sinc function is also called filter function.

☐ Triangle Function

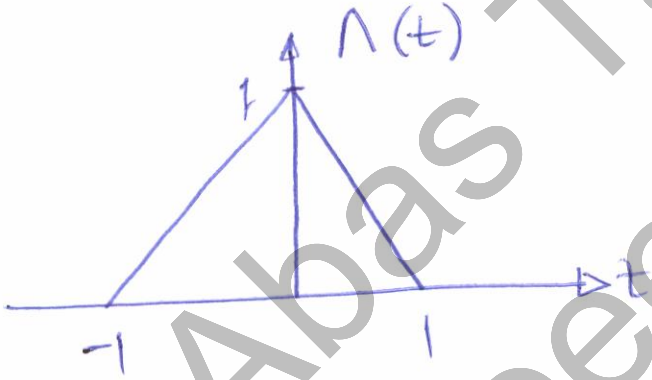
* Triangle function can be called as $Tri(t)$ or $\Lambda(t)$.

* Unit triangle function has unity area.



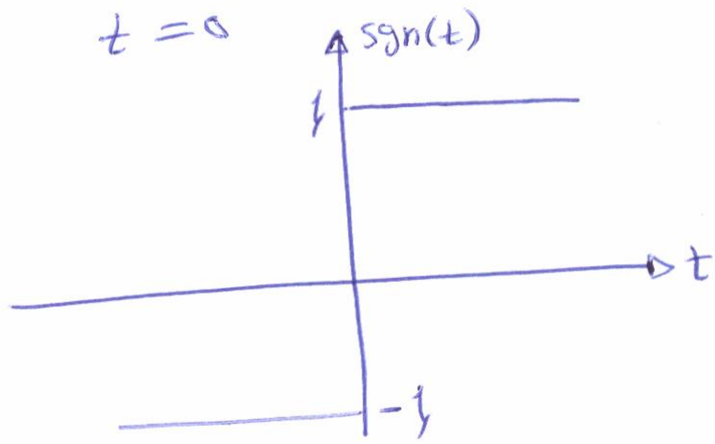
*Then, triangle function can be expressed mathematically as

$$tri(t) = \Lambda(t) = \begin{cases} t+1 & -1 \leq t \leq 0 \\ -t+1 & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{———} (68)$$



□ The Signum or sign function

$$sgn(t) = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \\ 0 & t = 0 \end{cases} \quad \text{———} (69)$$



EX. Calculate the energy of $g(t) = 3\,tri\left(\frac{t}{4}\right)$.

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |3\,tri(t)|^2 dt = 9 \int_{-\infty}^{\infty} tri^2(t/4)\, dt$$

But $tri\left(\frac{t}{4}\right) = \begin{cases} \frac{t}{4} + 1 & -1 \leq \frac{t}{4} \leq 0 = -4 \leq t \leq 0 \\ -\frac{t}{4} + 1 & 0 \leq \frac{t}{4} \leq 1 = 0 \leq t \leq 4 \\ 0 & \text{otherwise} \end{cases}$

also this function is even, However,

$$E_g = 9 \int_{-4}^{4} \left(1 - \left|\frac{t}{4}\right|\right)^2 dt = 18 \int_{0}^{4} \left(1 - \frac{t}{4}\right)^2 dt = 18 \int_{0}^{4} \left(1 - \frac{t}{2} + \frac{t^2}{16}\right) dt$$

$$= 18 \left[t - \frac{t^2}{4} + \frac{t^3}{48} \right]_0^4 = 24$$

Convolution Integral

* Finding the effect of a signal on another signal is called convolution.

* The convolution between two signals $f_1(t)$ and $f_2(t)$,

$$f_1(t) \circledast f_2(t) = \int_{-\infty}^{\infty} f_1(\tau)\, f_2(t-\tau)\, d\tau \qquad (70)$$

* Convolution integral is commutative:

$$f_1(t) \circledast f_2(t) = f_2(t) \circledast f_1(t) \qquad (71)$$

* Convolution integral is Distributive:

$$f_1(t) \circledast [f_2(t) + f_3(t)] = f_1(t) \circledast f_2(t) + f_1(t) \circledast f_3(t) \qquad (72)$$

* Convolution integral is Associative:

$$f_1(t) \circledast [f_2(t) \circledast f_3(t)] = [f_1(t) * f_2(t)] * f_3(t) \qquad (73)$$